Univ. of Illinois Due: 1 week Prof. Allen

Topics of this homework: Acoustics, Transmission line simulation, dB

This is a long homework. Do the Matlab/Octave problem (# 6) last! You may need more time for this one. Time estimates are given for solving the problem, but not for writing up the solution, which in many case may take longer than solving the problem. If you find that the actual time is significantly greater that my times, tell me (send me email or bring it up in class). For a demo version of this program download:

https://jontalle.web.engr.illinois.edu/537/M/tsline.m

1. Speech:

- (a) How many people talking, at 20 dB-SPL measured at 1 m are required to light a 60 watt light bulb? (5 min)
- (b) How many people are required if the are speaking at 60 dB-SPL (1 meter)? (1 min)
- (c) What is a phone?
- (d) How long is the average phone [ms]?
- 2. Anatomy of the vocal system:
 - (a) Hand-draw and label a picture of the Larynx. (10 min)
 - (b) What is the purpose of the Larynx? (2 min)
 - (c) How large is it? (2 min)
 - (d) If the lungs were a cube, how long would the side of the cube be in inches? (3 min)
 - (e) Work out the conversion factors and units required to convert mass, resistance and compliance, from MKS units to cgs? (10 min)

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C_t = 0.06 \text{ [m}^5/\text{Nt]} = [\mu\text{F}]

M_t = 0.04 \text{ [kg/m}^4] = [\text{Henrys}]

R_t = 4 \text{ [Nt-s/m}^5] = [\text{M}\Omega]
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- (f) Justify (derive/explain) the units of acoustic impedance for each of the elements in the previous question. For example, why does C_t have units of m⁵/Nt? (5 min)
- (g) What is the definition of Young's modulus, including the MKS units?
- (h) Define the following: (1-2 mins each)
 - i. Pharynx
 - ii. Hyoid
 - iii. Palate
 - iv. Mandible
 - v. velum
 - vi. velar
 - vii. Alveolar ridge (If necessary, look it up in the dictionary).
 - viii. viscosity
- 3. Wave equations:

(a) Show that the function f(x,t)

$$f(x,t) \equiv e^{-3(t-x/c)} \sin(2\pi 100(\sqrt{c}t - x/\sqrt{c})) U(ct - x)$$
 (1)

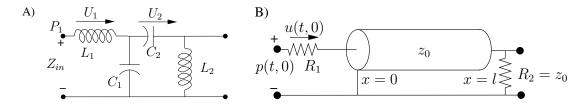
is a solution to the wave equation. U(t) is the unit step function. (2 min)

- (b) Explain where the formula $\gamma_0 P_0 = \rho_0 c^2$ comes from? Namely, derive this formula. (5 mins)
- (c) What year did D'Alembert derive his solution to the wave equation? (1 min)
- (d) What is the form of D'Alembert's solution? (1 min)
- (e) Webster Horn Equation:
 - i. Starting from the basic transmission line equations with an area function given by $A(x) = A_0 e^{2mx}$, derive the corresponding horn equation. I am not asking you to solve it, just write down the differential equation that describes wave propagation, when the area function is an exponential function. (7 min)
 - ii. What would you guess the form of the solution will be? (3 min max)

4. Reflectance:

- (a) Find (derive) the formula for the "input" impedance of a transmission line, having characteristic impedance $z_0(x, s)$, in terms of the reflectance. Define all the terms. Hint, I did this in class several times.

 (5 min)
- (b) Find the formula for the reflectance $\Gamma(s)$ in terms of the load impedance $Z_L(s)$ and the characteristic impedance z_0 if:¹
 - i. $Z_L(x, s) = r \, [\text{Nt-s/m}^5] \, (3 \, \text{min})$
 - ii. $Z_L(x,s) = 1/sC \text{ [Nt-s/m}^5\text{] (3 min)}$
 - iii. $Z_L(x,s) = r||sM|$ [Nt-s/m⁵] (5 min)
 - iv. Two transmission lines are in cascade, the first one having an area of 1 [cm²] and a second having an area of 2 [cm²], with lengths L_1 and L_2 respectively, terminated with a resistor $r = \rho c/A$, where $A = 2 \times 10^{-4}$ [m²]. Find R(x = 0, s).
 - v. What is the inverse Laplace transform of
 - A. H(s) = 1/(s+1)? Find h(t). (1 min)
 - B. $R(s) = \frac{Z-z_0}{Z+z_0}$ where Z=1 and $z_0=2$? Find r(t).
 - C. What if the line is 1 meter long and the speed of sound is 1 m/s?
 - D. H(s) = s/(s+1)? (1 min)
- 5. In the figure below, two circuits are shown, (A) and (B).



Analyze these circuits as follows:

 $^{^{1}}$ Note that r, C and M represent an acoustic resistance, compliance and mass. Namely they are positive constants.

- (a) Use a "traditional" analysis (define the impedances, and use formulas for series $[z = z_1 + z_2]$ and parallel $z_{12} = z_1 || z_2 = \frac{z_1 z_2}{z_1 + z_2}$ combinations), to obtain the input impedance $z_{in}(s)$, where s is the complex frequency $(s = \sigma + j\omega)$, defined in the *Laplace Transform*. (10 mins) This time only includes analysis, not the write-up.)
- (b) Use the ABCD (Transmission) matrix approach to find the input impedance of the same two circuits. (15 mins)
- (c) For circuit (B), if you set $R_1 = 0$, describe the poles and zeros of the input impedance of the transmission line.
- (d) Given the impedance matrix Z

$$\begin{bmatrix} z_{11}(s) & z_{12}(s) \\ z_{21}(s) & z_{22}(s) \end{bmatrix}$$
 (2)

find Y:

$$\begin{bmatrix} y_{11}(s) & y_{12}(s) \\ y_{21}(s) & y_{22}(s) \end{bmatrix}$$
 (3)

- 6. Write a Matlab[©] program to simulate a tube. Write up a summary description of your solution, with labeled figures, which includes a readable listing of the program. (2 hours) You can use MS word for you writeup, but I highly recommend LaTex when writing such a report, having equations and figures. There is some learning curve, but it pays off in the end, in my opinion.
 - (a) Write a Matlab program along the lines of what I discussed in class that simulates a impulse traveling along a transmission line with a sealed input end (glottis, on left) $(Z_g = \infty)$ and the output end (mouth, on right) terminated in a resistor having a resistance of $Z_m = r_m = 2z_0$, where z_0 is the characteristic impedance of the tube. Use a sampling frequency of $F_s = 44,100$ [Hz], and make the tube 10 samples long and assume c = 345 [m/s]. (1-3 hr, depending on how well you know Matlab). For a demo version of this program download:

http://hear.ai.uiuc.edu/ECE537//WWW/txline.m

- (b) Find the mouth (the output, on the right) and glottis (the input, on the left) velocity impulse response, and its spectrum (FFT of the impulse response).
- (c) When you plot an impulse response, use linear time-frequency plots. When you plot a frequency response, use log-log coordinates (log-frequency, log amplitude). Also only plot the positive frequencies from 100 [Hz] to 10 [kHz]. Use the best choice of frequency units (Hz vs kHz vs MHz vs GHz, but no GHz in acoustics, yet.) These are basic rules that if you need a good reason to violate.

Describe what you found. The writeup should also include an analysis of how you solve this problem, including the boundary conditions at the two ends of the tube, the length of the tube, etc. (30-60 min)